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ABSTRACT

Several computer programs were developed to solve various versions of the quadratic capacitated facilities location problem. Matrices, which represent various business costs, are defined for the factors of sites, facilities, customers, commodities, and production units. The objective of the program is to find an optimization matrix for the lowest cost, given the restrictions of the problem. The algorithms, COMPAT and SWITCH, are devised to solve portions of the problem with the subroutines, CHANGE and MUNKRES. These programs are tried on several different problems, and the different results are compared. Suggestions for future research, particularly using Gilmore's branch and bound technique, are made. (WH)

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SOLVING THE QUADRATIC CAPACITATED FACILITIES LOCATION PROBLEM BY COMPUTER

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When there are capacity restrictions on some or all of the facilities and when the cost of interfacility interaction is considered, the resulting problem of site selection is a Quadratic Capacitated Facilities Location Problem, which is formulated and discussed in this paper. It is shown that The Capacitated Facilities Location Problem, the Simple Facilities Location Problem, and the Quadratic Facilities Location Problem are but special cases of this more general problem.

Mathematical Formulation of The Quadratic Capacitated Facilities Location Problem

Consider a set of a suppliers furnishing raw materials to a given set F of facilities (sources) $F \leq d$ d = number of possible facilities to be located, where these materials are transformed into a single finished commodity and distributed to n customers (sinks). Each customer j has a demand for d_j units of the commodity.

There is a fixed cost matrix F with typical entry. $f_{ik} \geq 0$, expressing the fixed cost of establishing a facility i at location k. If there are d possible facilities, m possible facility locations, and n customers, m d then i = 1,...,d and k = 1,...,m. These costs can be interpreted as construction costs, land purchase costs, set up cost, machinery purchase costs or other fixed costs. If plant i is not established at location k, $f_{ik} = 0$, otherwise $f_{ik} > 0$.

x_{ikj} is the amount produced in facility i established at location or site k and shipped to customer j.

A decision variable y_{ik} is introduced such that $y_{ik} = 1$ if plant i is established at location k and $y_{ik} = 0$ otherwise.

In addition to the fixed costs, there are the variable costs or cost per unit of commodity produced which are assumed constant regardless of the number produced. These costs combine:

1. the purchase cost of raw materials per unit of commodity produced, assuming no quantity discounts.

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2. the sum of distribution cost for raw materials (per unit of finished commodity) from all s suppliers to facility i at location k.

3. the transportation costs per unit of commodity from facility i at location k to customer j and

4. any other costs which vary linearly with the number of units produced, expressed as cost per unit of commodity.

The combination of these costs are hereafter referred to as the transportation costs and are specified by a matrix T of unit costs with typical entry t_{ikj}, which expresses the variable cost of producing a unit of commodity in facility i established at location k and shipping it to customer j.

There is a matrix I with typical entry i_{ir} which expresses the interfacility flow from facility i to facility r.

A matrix D with typical entry d_{ks} expresses the variable cost (distance) of shipping a unit of commodity from location k to location s.

A matrix u with typical entry u_{ik} expresses the capacity restriction of facility i in location k.

The problem reduces to that of determining the matrix Y with typical element y_{ik} where

$$y_{ik} = \begin{cases} 1 \text{ if facility i is established at location k} \\ 0 \text{ otherwise} \end{cases}$$

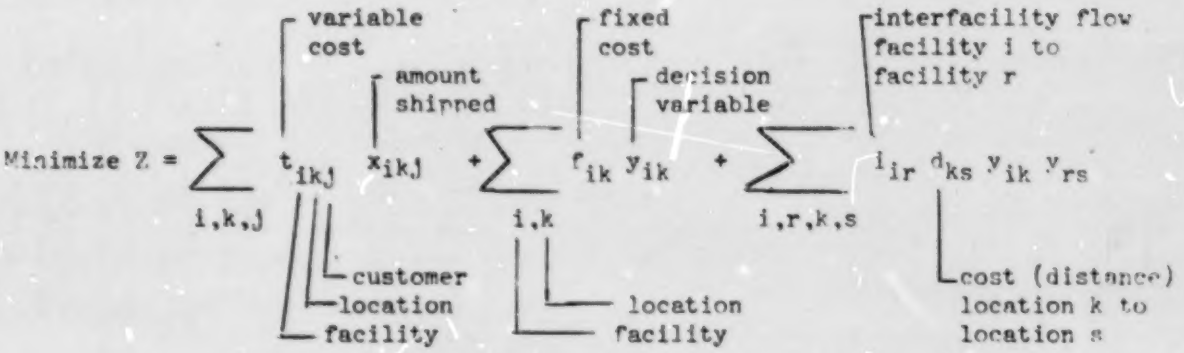
such that the total cost is a minimum. This minimization problem is subject to the constraints, numbered 1 through 10 in the following summarized problem formulation, where Z is the total cost to be minimized.

Comparisons of Variations of the Problem

If the capacity constraints 7 and 8 are removed from this problem, the resulting problem is the Quadratic Facility Location Problem.

The removal of the quadratic term in the objective function, together with constraints 9 and 10 which accompany this term, results in the Capacitated Facility Location Problem.

QUADRATIC CAPACITATED FACILITY LOCATION PROBLEM



Facility i = i,...,d location k = 1,...,m customer j = 1,...,n

SUBJECT TO:

1. $\sum_j x_{ikj} = x_{ik}$ total production of facility i in location k

2. $\sum_{i,k} x_{ikj} = d_j$ satisfy customer demand

3. $f_{ik} \geq 0$ non-negative set up costs of facility i in location k

4. $x_{ikj} \geq 0$ non-negative flow from facility i at location k to customer j

5. $t_{ikj} \geq 0$ variable cost per unit of commodity produced in facility i at location k and shipped to customer j

6. $y_{ik} = \begin{cases} 1 & \text{if facility i is established at location k} \\ 0 & \text{otherwise} \end{cases}$

7. $0 \leq x_{ik} \leq u_{ik}$ capacity restriction of facility i in location k (min. of restrictions of facility and/or location capacities)

8. $\sum_j d_j \leq \sum_{i,j} u_{ik} y_{ik}$ total demand $\leq$ total capacity

9. $d_{ks} \geq 0$ non-negative transportation cost (distance) from location k to location s

10. $y_{rs} = \begin{cases} 1 & \text{if facility r is established at location s} \\ 0 & \text{otherwise} \end{cases}$

The removal of the quadratic term, together with the accompanying constraints 9 and 10 and also the removal of the capacity constraints 7 and 8, results in the simple facility location problem.

The following table and schematic diagram give an overview of these relationships between the Quadratic Capacitated Facility Location Problem and the three special cases:

1. the Quadratic Facility Location Problem (Quadratic Assignment Problem).
2. the Capacitated Facility Location Problem.
3. the Simple Facility Location Problem (Linear Assignment Problem).

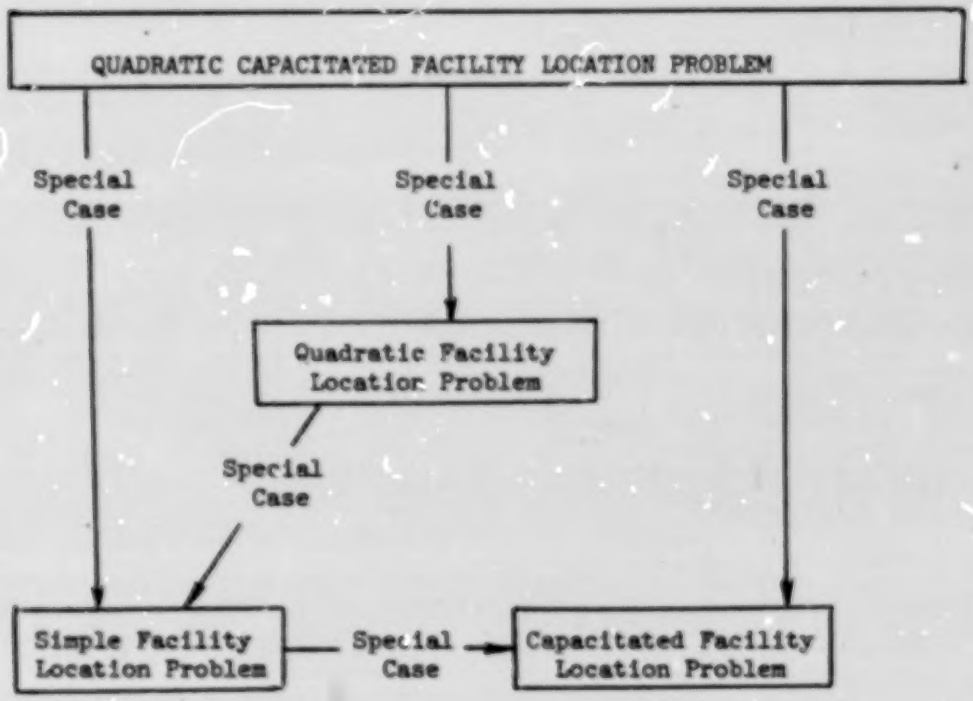
Methodology

The Capacitated Facilities Location Problem and the Quadratic Capacitated Facilities Location Problem are reformulated as quadratic assignment problems. The resulting Quadratic Assignment Problem is partitioned into a smaller quadratic assignment problem and a linear assignment problem. A new algorithm, COMPAT, is devised to solve both portions of the partitioned quadratic assignment problem and coordinate them into a single solution. Another new Algorithm, SWITCH, is devised to solve both portions of the partitioned quadratic assignment problem and coordinate them into a single solution. The results of solutions secured by both the COMPAT and SWITCH algorithms are compared. An evaluation of the two newly developed algorithms is made and suggestions are made for possible further extensions of the techniques developed.

The COMPAT Algorithm partitions the problem into the linear problem of assigning customers to customer locations, and the quadratic assignment problem (which is not necessarily purely quadratic) of assigning facilities to facility locations. The facilities assignment portion is purely quadratic if the set of facilities are all interconnected by flows.

The original quadratic problem may be a quadratic capacitated facilities location problem, a quadratic facilities location problem, or simply a capacitated

Problem	Capacity Restrictions on Facility Size	Interfacility Flow Considered
Quadratic Capacitated Facility Location	YES	YES
Simple Facility Location	NO	NO
Quadratic Facility Location	NO	YES
Capacitated Facility Location	YES	NO



facilities location problem which has been cast into the proper form. Problems which may be cast into the appropriate form include: the warehouse location problem, the backboard wiring problem, and the traveling salesman problem.

The Subroutine CHANGE

The Subroutine CHANGE is a basic interchanging algorithm which interchanges the assignments of two members, evaluates the cost of the new assignment and accepts the new assignment if it is better. The interchange of another pair of facility assignments is made, evaluated and accepted or rejected. The process is repeated until no further interchange yields an advantageous solution.

It is generally acknowledged that the interchanging or trading procedure is the best available solution procedure for the quadratic assignment problem.

The Subroutine MUNKRES

The subroutine MUNKRES solves a linear assignment problem in an efficient and optimal manner. It is a variation of Kuhn's Hungarian Assignment Algorithm.

Munkres' Linear Assignment Algorithm is an exact (or optimal) algorithm. With a given facility assignment the assignment of customers is a linear assignment problem. Therefore, for a given facility assignment, the COMPAT Algorithm makes an optimal customer assignment. However, the facility assignment was made by a suboptimal interchanging procedure and hence the assignment of customers only approximates a linear assignment problem. This suggests using a suboptimal procedure to solve this portion of the problem.

The SWITCH Algorithm

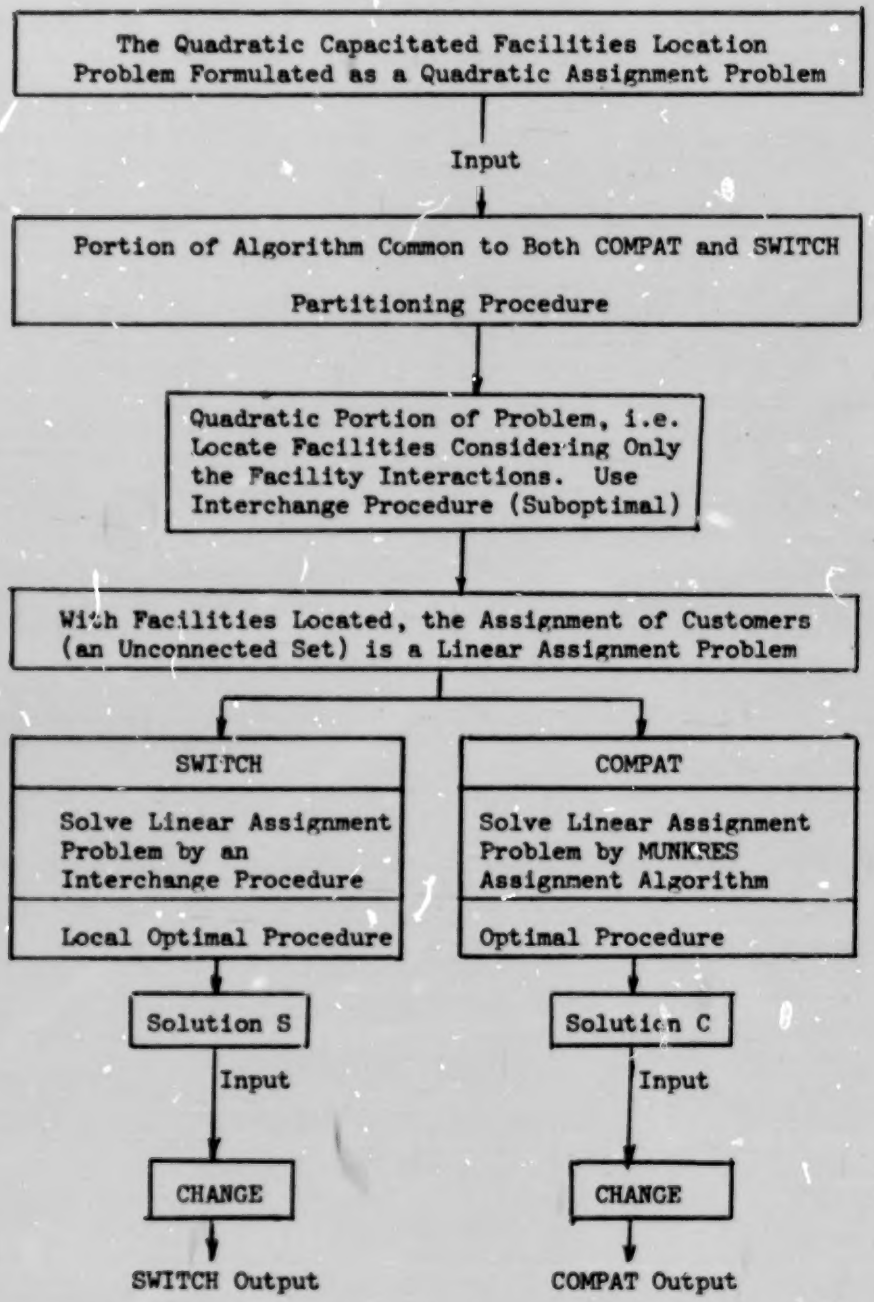
The SWITCH Algorithm replaces the optimal subroutine MUNKRES used in the COMPAT Algorithm with the suboptimal subroutine CHANGE.

Analysis of Results and Suggestions for Future Research

The COMPAT and SWITCH Algorithms were used to solve several problems.

A problem will be designated by three numbers, viz:

1. The first number designates the number of unit demand customers.


The Quadratic Capacitated Facilities Location Problem Formulated as a Quadratic Assignment Problem
Input
Portion of Algorithm Common to Both COMPAT and SWITCH
Partitioning Procedure
Quadratic Portion of Problem, i.e. Locate Facilities Considering Only the Facility Interactions. Use Interchange Procedure (Suboptimal)
With Facilities Located, the Assignment of Customers (an Unconnected Set) is a Linear Assignment Problem
SWITCH
Solve Linear Assignment Problem by an Interchange Procedure
Local Optimal Procedure
COMPAT
Solve Linear Assignment Problem by MUNKRES Assignment Algorithm
Optimal Procedure
Solution S
Solution C
Input
Input
CHANGE
CHANGE
SWITCH Output
COMPAT Output

2. The second number designates the total number of unit facilities to be established including any dummy facilities.

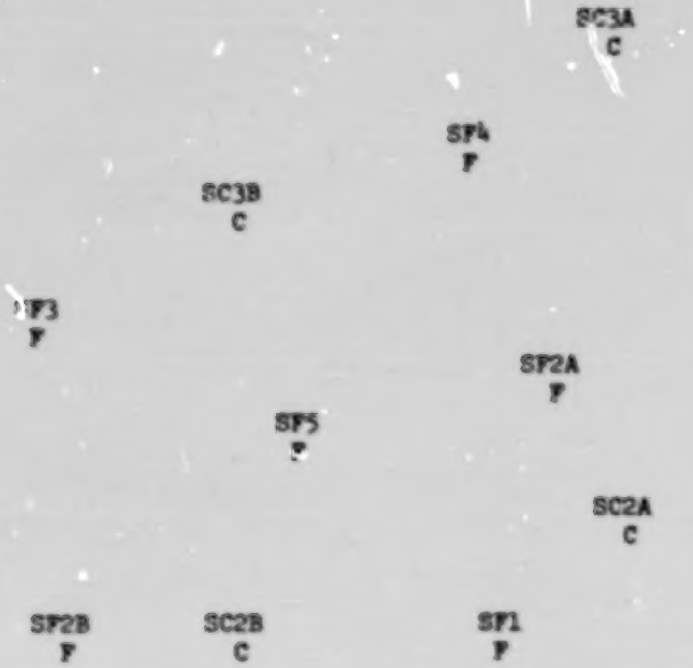
3. The third number designates the total number of flows being considered.

Different problems may consider the same numbers of customers, facilities and flows. Therefore, the three numbers may be followed by a letter. Thus, 10,27,29A and 10,27,29B would designate two completely different problems.

A solution is designated by a C or an S prefixing the problem designation, the C or S prefix indicating whether the solution was obtained by the COMPAT or SWITCH Algorithm. In any particular problem, customers or facilities will be designated by C or an F followed by a number representing the customer demand or facility capacity. Thus, F10A and F10B represent two facilities, each having a capacity of 10 units, and SC6A means the site of customer C6A having a demand of 6 units.

Problem 10,27,29A

Four facilities are to be established in some subset of 6 available sites to serve 4 customers. These sites are tabulated together with the planar coordinates assumed for each site.



	IX	IY
SC3A	25	27
SC3B	11	22
SC2A	25	11
SC2B	11	7
SF5	13	14
SF4	20	23
SF3	4	18
SF2A	22	17
SF2B	5	7
SF1	21	7

It is desired to establish 4 facilities subject to the fixed and production costs, interfacility flows and available site locations as follows:

Facility	ICOST
F4	15
F3	15
F2	15
F1	15

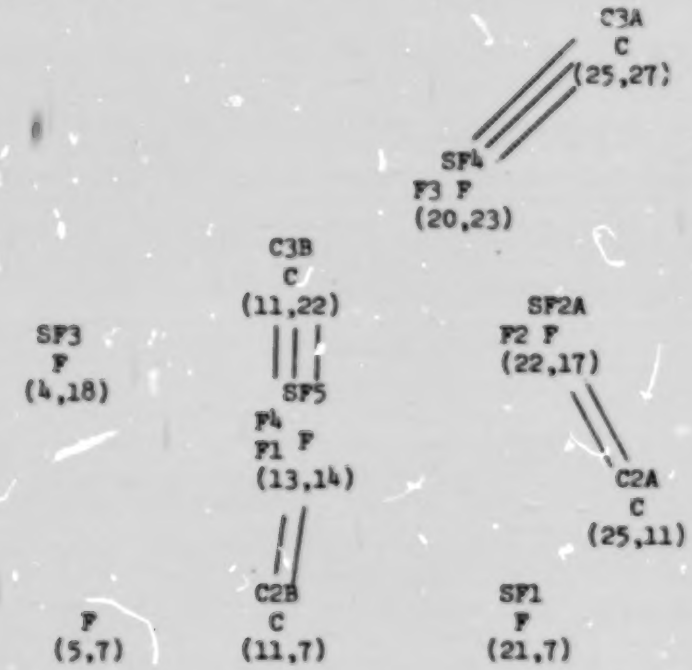
and

	F4	F3	F2	F1
F4		.3	.1	.2
F3			.2	.1
F2				0
F1				

A SUMMARY OF PROBLEM 10,27,29A

The Solutions C,10,27,29, and S,10,27,29

The COM'AT and SWITCH Algorithms yielded identical solutions for this problem. The common solution is best shown below. The cost was 906. The final stage of the SWITCH Algorithm started with a cost of 906 and after 271 iterations had 75 advisable interchanges.



THE COMMON SOLUTIONS S,10,27,29, AND C,10,27,29

Problem 24, 81,95

Locate 4 facilities with capacities of 8,7,5 and 4 to serve a total customer demand of 24. There are 7 available facility sites with capacities of 10,10,10,10,8,5,4.

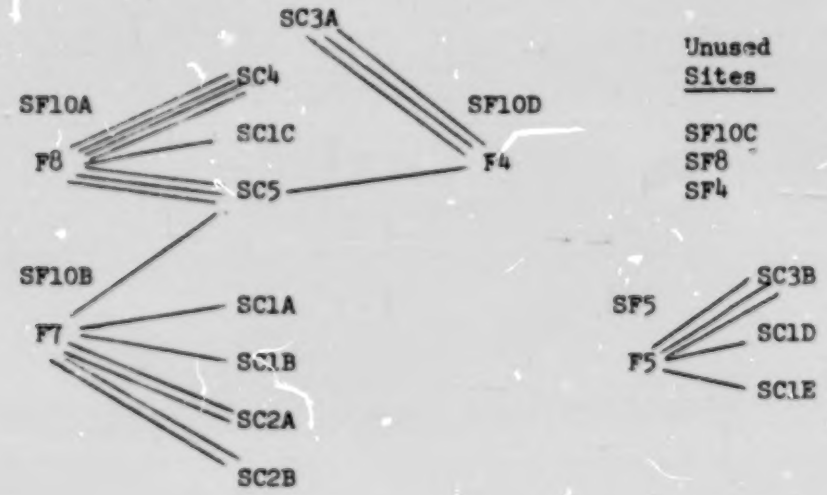
The following table summarizes this portion of the problem:

Customer Location	Code (with demand)	Available Facility Site	Code (with capacity)
Portland, Ore.	SC5	St. Paul, Minn.	SF10A
Spokane, Wash.	SC4	Kansas City, Mo.	SF10B
Salt Lake City, Utah	SC3A	Seattle, Wash.	SF10C
Philadelphia, Pa.	SC3B	Salt Lake City, Utah	SF10D
Wichita, Ka.	SC2A	Washington, D.C.	SF8
El Paso, Tex.	SC2B	Memphis, Tenn.	SF5
San Antonia, Tex.	SC1A	Dallas, Tex.	SF4
Pittsburgh, Pa.	SC1B		
New York, N.Y.	SC1C		
Richmond, Va.	SC1D		
Louisville, Ky.	SC1E		

It is desired to locate facilities F8, F7, F5, and F4 subject to the costs of interfacility flows expressed in the following matrix as well as the fixed plus production costs given in the attached table.

	F8	F7	F5	F4	ICOST/unit
F8		2.2	1.8	1.1	25
F7			1.3	.9	30
F5				.4	40
F4					38

It should be observed that the consideration of interfacility flow costs requires a quadratic problem formulation and solution. The solution S,24,81,95A, shows that the customer at Portland, Oregon, SC5, should be serviced by three different facilities. This illustrates the capacitated nature of the solution obtained.



SCHEMATIC DIAGRAM OF SOLUTION S,24,81,95

Variations of the Algorithms and Results Obtained

Variations of the COMPAT and SWITCH Algorithms were tried on the problems 10,27,29 and 24,81,95. All variations of the procedures used the same initial solutions.

The basic problem being solved is a quadratic capacitated facilities location problem. The COMPAT and SWITCH Algorithms partition the overall problem and apply solution techniques to the subproblems before attacking the overall problem utilizing these subproblem solutions. Therefore, for comparison purposes, the unpartitioned problems were solved by the interchanging procedure with the following results.

		Objective Value		
Problem	Convergence	COMPAT	SWITCH	Unpartitioned
10,27,29,A	YES	906	906	1400
24,81,95,A	YES	2511	2511	3278

The assignment of the facilities were the same in all cases but the solution of the unpartitioned problem by CHANGE converged to a solution having a non-optimal assignment of customers. This indicates that since an optimal algorithm for customer assignment is available, MUNKRES, it should be the final step in any problem solution if possible. In any case, the problem should be partitioned until an optimal algorithm is developed for the overall problem. In the smaller problem 10,27,29, a savings of over 35 per cent was realized by first partitioning the problem and then utilizing the CHANGE Algorithm. Partitioning yielded a savings of over 23 per cent, with the larger problem 24,81,95.

In one variation, the unpartitioned problem 10,27,29 was solved by CHANGE yielding an objective value of 1400. This was followed by MUNKRES which reduced the value from 1400 to 906 and gave the same solution as the COMPAT and SWITCH solutions.

Suggestions for future research

1. Apply Gilmore's branch and bound technique (or some similar optimizing technique) to the set of facilities.
2. With the set of facilities thus optimally located, formulate the linear assignment problem of customer assignment.

3. Optimally assign the customers by MUCKRES or some other optimal procedure, such as Gilmore's branch and bound.

4. Since optimizing the portions of a partitioned problem does not guarantee an optimal solution of the composite problem, combine the subproblem solutions to form an overall solution, which must be calssified as suboptimal.

5. Use the combined (suboptimal) solution obtained in 4 as an initial input solution to a good suboptimal technique such as CHANGE. (For clarity, call this solution GILPART for GILmore with PARTitioning.)

6. Use Gilmore's branch and bound technique on a small unpartitioned problem to obtain an optimal solution. (Again for clarity, call the solution GILWHOLE.)

7. Compare the GILWHOLE, GILPART, COMPAT and SWITCH results to the same small problem.

Drawbacks--Gilmore's branch and bound technique is computationally very time consuming and, therefore, would be restricted to very small problems. To increase the size of the problem which might be considered, an omission of GILWHOLE could be made.

The combined use of Gilmore's branch and bound technique with an interchanging technique would be new, as would be the combined use of Gilmore's branch and bound technique with an assignment algorithm such as MUNKRES.

GILPART G	GILPART M
1. Branch and bound on facilities	1. Branch and bound on facilities
2. Branch and bound on customers	2. MUNKRES on Customers
3. SWITCH on combined 1 and 2	3. SWITCH on combined 1 and 2

It is realized that parts 1 and 2 which form the input solution to 3 are both optimal solution procedures. However, they may generate alternate optimal solutions in part 2, thereby forming alternate suboptimal input solutions to SWITCH in part 3.

A SUMMARY OF SOME COMPUTATIONAL RESULTS ON THE CDC 6400 COMPUTER

Problem	Solution	Converge	Feasible	Objective Value	CP Time	Field Required
10,27,29	SWITCH	YES	YES	906	2.893	$36,500_8$
	COMPAT	YES	YES	906	6.004	$39,500_8$
	Unpartitioned by CHANGE	YES	YES	1,400	3.502	$36,500_8$
24,64,95	COMPAT	YES	NO	218,076,355	50.427	$40,700_8$
24,81,95	SWITCH	YES	YES	2,511	63.495	$36,500_8$
	COMPAT	YES	YES	2,511	65.328	$41,100_8$
	Unpartitioned by CHANGE	YES	YES	3,278	74.063	$36,500_8$
96,229,456A	SWITCH	YES	NO	143,844,498	2013.664	$37,200_8$
	COMPAT					$140,000_8$
96,297,456	SWITCH	YES	NO	5,575,812	3600.186	$37,500_8$
96,297,456	COMPAT					140.000_8

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